CHAPTER 22 Electric Field

Answers to Understanding the Concepts Questions

1. The moving truck picks up electric charge as it moves. Rubber tires are good insulators; the charge will not automatically flow to the ground. There is danger that when enough charge builds up, a breakdown can occur with the formation of a spark, and such a spark is extremely dangerous when gasoline is present. For just this reason, tires are made today with materials that conduct well, and a dragging chain is no longer necessary.

2. The direction of the electric field is tangent to the electric field lines. If two field lines intersect then there are two possible tangents at the intersection, and yet the actual electric field can point only at one direction.

3. The introduction of a gravitational field $\vec{g} = \vec{F}_g/m$ is indeed useful for the same reasons that the introduction of an electric field is useful. The field resembles that of the electric field in that in the absence of matter (charge) the field lines are continuous, and their density represents the strength of the field. It differs in that mass comes in only one sign: gravity is a uniquely *attractive* force, so that field lines have only one end on matter. The other end must be at infinity, since there is no mass of opposite sign for the line to attach to. In other words, there is no analogue of an overall neutral charge distribution, in which lines start in part of the distribution and end elsewhere.

4. When placed against a metal wall, the excess charge on the balloon would induce a buildup of opposite charge on the wall, causing the balloon to be attracted to it. When placed against an insulating wall the excess charges on the balloon would polarize the molecules on the wall, and the resulting attractive force would also make the balloon stick to the wall.

5. The electric field lines emerge from positive charges and end up at negative charges. The density of the field lines indicates the strength (magnitude) of the field, which is proportional to the charge that produces the field. Since five times the field lines leave one charge (q_1) as end up at the other (q_2), q_1 is positive and q_2 is negative, and $q_1/q_2 = -5$.

6. Not really, since we may think of a negative charge $-Q$ distributed uniformly over a spherical surface at infinity to accompany our single positive point charge Q. The charge density everywhere is zero, so that this depiction has no practical consequence other than the satisfying notion that the universe involving single charges is still electrically neutral.

7. To the left of q_1 the net force is to the left (nonzero), and in between q_2 and q_3 the net force is to the right. There are two points where E = 0, one is in between q_1 and q_2 (-2 cm $< x <$ +4 cm), and the other is somewhere to the right of q_3 ($x >$ 10 cm).

8. The magnitude of the field of a dipole decreases with the distance r between the center of the dipole and the point of interest. In fact it can be shown that $E(r)$ goes like $1/r^3$. While $E(r)$ is nonzero for any finite r, as r approaches infinity $E(r)$ approaches zero.

9. We know that because the electric field above Earth's surface points downward, toward Earth.

10. In order to visualize a sphere with an induced dipole moment, think of the induction of a positive charge $+Q$ at the sphere's north pole and a negative charge $-Q$ at the south pole in response to an external field oriented along the north-south axis. Suppose that we now suddenly change the external field so that it is now perpendicular to the north-south axis. The charges will move in response so that now the dipole moment is oriented along the direction of the new external field. But since these charges are not attached to the conductor — they move freely on the conducting surface — their motion does not induce a rotation of the sphere. With a long rod, the situation is different. Even if the charges are free to move within the conductor, the shape of the conductor itself restricts the movement of the charges. Thus there will be equal and opposite forces on the two ends, tending to rotate the rod. At the same time, the original inducing field is now gone, and the charges rush back to each other under the influence of the coulomb forces between them. Whether there is an actual motion of the rod depends on how rapidly the charges move back together compared to how rapidly the new field acts.

11. The net electric charge present on the comb causes the molecules in the paper to polarize; so the region that's closer to the comb has a net charge that's opposite in sign to that of the comb, resulting in a net attraction.

12. In principle, yes it can. The field lines of an electrostatic field can only begin and end where there is a charge present. For example, suppose there is an isolated positive charge from which field lines emerge. These field lines will not end unless there is a negative charge. If no other charges are present the field lines will extend to infinity, even though the density of the field lines becomes zero now that they are spread out infinitely far from each other, meaning that the magnitude of the electric field there is zero.

13. The total charge of the water molecule is zero. The charge distribution shown in the figure suggests that the electric field is that of two dipoles touching at one end. The superposition of two electric dipole fields is again an electric dipole field (they both fall as $1/r^3$), except under very special circumstances in which there is a cancellation, so that only the $1/r^4$ terms are left. This is not the case here.

14. A small hole can be drilled on the negatively charged receptor plate to allow protons to pass through.

15. The density of the field lines (number of lines per unit cross-sectional area) represents the magnitude of the electric field. Suppose there are a total of N field lines which emerge from a positive charge. A distance r from the charge, these field lines are evenly distributed over a spherical surface of radius r, so the density of the field lines there is $N/A = N/4\pi r^2$, which is proportional to $1/r^2$, an accurate representation of the r-dependency of the magnitude of the electric field. If the electric field changes with r in any other power, then the field line density, which always goes like $1/r^2$, would no longer represent the magnitude of the field.

16. The velocity field has features common to the electric field. Sources (like faucets) correspond to positive charges, and sinks (like drains) correspond to negative charges. The velocity field is represented by a vector at every point in space, just like the electric field. The major difference is that in a liquid there is something that actually moves along the lines (look back at Chapter 16), whereas the electric field lines do not represent motion except in the sense that a test charge would accelerate along the tangent of a field line. The electric field is thus more like an "acceleration field," something which is of little interest in the study of fluids.

17. An electric field line can only end up at a negative charge. It is therefore impossible to construct such an arrangement for the electric field lines to be directed into a point where no charge is present.

18. From the figure we can see that the forces will align the small dipole in such a way that the attraction is maximized, or such that the potential energy is minimized. In other words, the small dipole will align its electric dipole moment to be antiparallel to the large fixed dipole's electric dipole moment.

19. This configuration can indeed be thought of as two electric dipoles of equal strength pointing at opposite directions. It is an example of an electric *quadrupole*. The field produced by this setup is not exact zero. This is because the two dipoles, while equal in magnitude and opposite in orientation, are slightly displaced from one another so their fields do not completely cancel out, even though the net field does drop rapidly as the distance r from the origin (as $1/r^4$, as can be shown).

20. The positive charges from these dipoles form an infinite, uniform sheet of charge, while the negative charges form its own sheet, parallel to the first one, with exactly the opposite density of charge. The net field is the superposition of those from the two sheets, so it must be zero.

21. Both the force of gravity and the electrical force are independent of the height. If the gravitational force of attraction is stronger than the repulsive force, the pellet will fall down, albeit with an acceleration smaller than that due to gravity. If the repulsive force is stronger, then the particle will accelerate away from the plate, and go upwards.

22. Two forces are exerted on the pellet: the electrostatic repulsive force from the sphere, up; and the gravitational force, down. The motion of the pellet depends on the relative strengths of these two forces. If the charge on the sphere is relatively weak then the electrostatic repulsion cannot prevent the pellet from colliding with the top of the sphere. If the electric repulsion is relatively strong then the pellet will not be able to reach the surface of the sphere. Rather, it is accelerated (by gravity) toward the sphere until it reaches the equilibrium position (where the electrostatic repulsion is equal to its weight), then decelerates while continuing to move downward toward the sphere. Eventually it comes to a momentary stop due to the strong repulsion of the sphere, and then starts to move back up, accelerating towards (and past) the equilibrium point before decelerating to a stop. Afterwards the motion repeats itself, with the pellet oscillating up and down above the sphere, reaching the greatest speed upon passing the equilibrium position.

23. The electric field at the origin is now dominated by the charge q_1 located at $x_1 = -1$ mm, as q_1 is so much closer to the origin than any other charge. As a good approximation we may just calculate the field due to q_1 and neglect those due to q_1 and q_2.

24. When the distance between a point and the surface of a sphere is much less than the radius of the sphere, the sphere can be approximated as a plane from the perspective of that point. Such is the case of Example 22-10.

Solutions to Problems

1. The displacement $\vec{r}$ from the charge is shown in the diagram. We find its magnitude from

$$r^2 = x^2 + y^2$$
$$= (3\text{ cm})^2 + (4\text{ cm})^2, \text{ which gives } r = 5\text{ cm}.$$

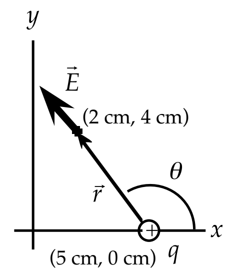
We find its direction from

$$\tan\theta = y/x$$
$$= (4\text{ cm})/(-3\text{ cm}) = -1.33,$$

which gives $\theta = 127°$.

The electric field at (2 cm, 4 cm) is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$
$$= \left(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(5\times10^{-6}\ \text{C}\right)}{\left(5\times10^{-2}\ \text{m}\right)^2}\left(\cos\theta\,\hat{i}+\sin\theta\,\hat{j}\right)$$
$$= 1.8\times10^7\left(-0.60\hat{i}+0.80\hat{j}\right)\text{N/C}.$$



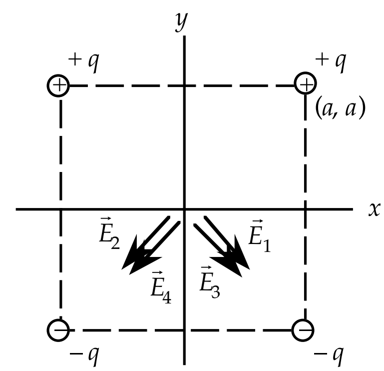
2. Because the origin is equidistant from the equal charges, the electric fields will have the same magnitude:

$$E_i = (1/4\pi\varepsilon_0)[q/(a\sqrt{2})^2] = (1/4\pi\varepsilon_0)(q/2a^2).$$

The electric fields are shown in the diagram.

From the symmetry, we have

$$\vec{E} = \sum E_{iy}\,\hat{j} = -4[(1/4\pi\varepsilon_0)(q/2a^2)]\cos 45°\,\hat{j}$$
$$= \boxed{-(1/4\pi\varepsilon_0)(q\sqrt{2}/a^2)\,\hat{j}.}$$



3. Because we can treat the nucleus as a point charge, the field will be radial:

$$\vec{E} = (1/4\pi\varepsilon_0)(q/r^2)$$
$$= [(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(79)(1.60\times10^{-19}\ \text{C})/(1\times10^{-9}\ \text{m})^2] = \boxed{(1.14\times10^{11}\ \text{N/C}).}$$

The force on an electron is

$$\vec{F} = q\vec{E} = -e\vec{E}$$
$$= -(1.60\times10^{-19}\ \text{C})(1.14\times10^{11}\ \text{N/C}) = \boxed{-(1.82\times10^{-8}\ \text{N})\ \text{(toward the nucleus)}}.$$

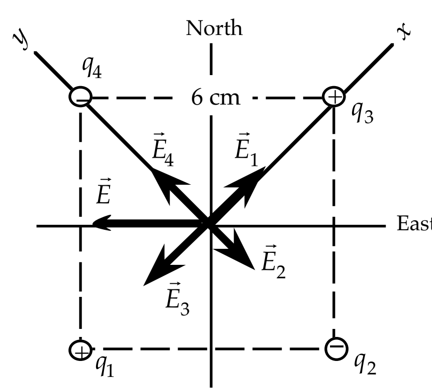
4. Because the electric field from each of the charges is along a diagonal of the square, we choose the xy-coordinate system in the following way: the direction from -2μC to -5μC is the positive x-axis (east), and the direction from $+7\ \mu$C to $+3\mu$C is positive y-axis (north). We have

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$
$$= (k/r^2)(5\mu\text{C} - 2\mu\text{C})\,\hat{i} + (k/r^2)(7\mu\text{C} - 3\mu\text{C})\,\hat{j},$$

where $r = a/\sqrt{2}$, $a = 0.040$ m, and $k = 9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

The result is

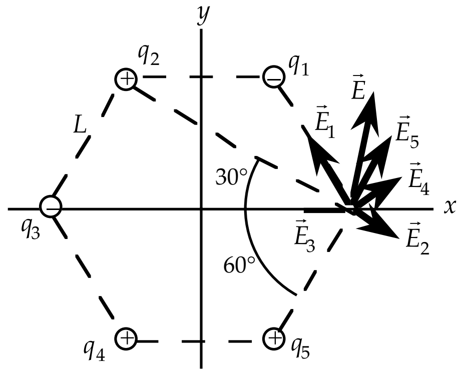
$$\vec{E} = [(3.4\times10^7\,\hat{i}) + 4.5\times10^7\,\hat{j})]\ \text{N/C}$$
$$= 5.6\times10^6\ \text{N/C, } 53° \text{ above the } x\text{-axis, i.e.,}$$
$$\vec{E} = \boxed{5.6\times10^6\ \text{N/C, } 53° \text{ north of east}}.$$



5. For a regular hexagon, we have the angles shown. The edge is $L = 10$ cm. For the distances from the charges we have

$$r_1 = r_5 = L = 10 \text{ cm};\ \ r_2 = r_4 = 2L\cos 30° = 17.3 \text{ cm};\ \ r_3 = 2L = 20 \text{ cm}.$$

We take advantage of the symmetry of the charges to simplify the vector addition of the individual fields. Because $q_1 = -q_5$, and $r_1 = r_5$, the magnitudes of $\vec{E}_1$ and $\vec{E}_5$ will be equal. Their resultant will be in the y-direction:



$$\vec{E}_1 + \vec{E}_5 = 2[(1/4\pi\varepsilon_0)q_1/r_1^2]\sin 60°\,\hat{j}$$
$$= 2[(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\times10^{-6}\ \text{C})/(0.10\ \text{m})^2]\sin 60°\,\hat{j}$$
$$= (3.12\times10^6\ \text{N/C})\,\hat{j}.$$

Because $q_2 = q_4$, and $r_2 = r_4$, the magnitudes of $\vec{E}_2$ and $\vec{E}_4$ will be equal. Their resultant will be in the x-direction:

$$\vec{E}_2 + \vec{E}_4 = 2[(1/4\pi\varepsilon_0)q_2/r_2^2]\cos 30°\,\hat{i}$$
$$= 2[(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\times10^{-6}\ \text{C})/(0.173\ \text{m})^2]\cos 30°\,\hat{i}$$
$$= (1.56\times10^6\ \text{N/C})\,\hat{i}.$$

For $\vec{E}_3$ we have

$$\vec{E}_3 = -[(1/4\pi\varepsilon_0)q_3/r_3^2]\,\hat{i}$$
$$= -[(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4\times10^{-6}\ \text{C})/(0.20\ \text{m})^2]\,\hat{i} = -(0.90\times10^6\ \text{N/C})\,\hat{i}.$$

The resultant electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5 = [(0.66\times10^6\,\hat{i}) + 3.12\times10^6\,\hat{j})]\ \text{N/C}$$
$$= \boxed{3.19\times10^6\ \text{N/C, 78° above the } +x\text{-axis}}.$$

6. (a) The electric field of q_1 will be away from q_1 with a magnitude

$$E_1 = (1/4\pi\varepsilon_0)(q_1/r_1^2)$$
$$= (9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.5\times10^{-6}\ \text{C})/(0.22\ \text{m})^2$$
$$= \boxed{2.8\times10^5\ \text{N/C}}\ \text{away from } q_1.$$

(b) This field produces an attractive force on q_2:

$$F_2 = q_2E_1 = (3.5\times10^{-6}\ \text{C})(2.8\times10^5\ \text{N/C}) = 0.98\ \text{N toward } q_1.$$

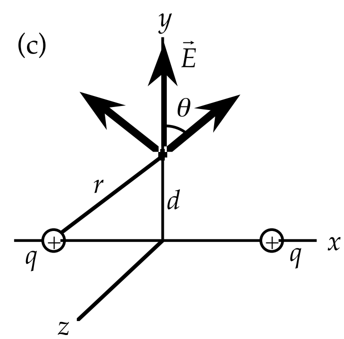
(c) At the midpoint, both fields will be toward q_2. The resultant field is

$$E_1 + E_2 = [(1/4\pi\varepsilon_0)q_1/r_1^2] + [(1/4\pi\varepsilon_0)q_2/r_2^2]$$
$$= (9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\{[(1.5\times10^{-6}\ \text{C})/(0.11\ \text{m})^2] + [(3.5\times10^{-6}\ \text{C})/(0.11\ \text{m})^2]\}$$
$$= \boxed{3.7\times10^6\ \text{N/C}}\ \text{toward } q_2.$$

7. (a) With the charges on the x-axis, the electric fields produced by the charges will have the same magnitude and point in the $-x$-direction. The resultant field will be

$$\vec{E} = 2(1/4\pi\varepsilon_0)[q/(\ell/2)]^2(-\hat{i}) = -(1/4\pi\varepsilon_0)(8q/\ell^2)\,\hat{i}.$$

(c)



(b) The fields produced by the charges will have the same magnitude and point in opposite directions. The resultant field will be $\vec{E} = \boxed{0}$.

(c) We take a representative point on the y-axis. From the diagram, we see that the electric fields produced by the charges will have the same magnitude, and the resultant field will point away from the origin. If we call the distance from the origin d, we have

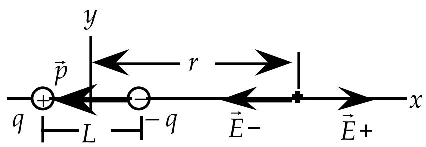
$$E = 2(1/4\pi\varepsilon_0)(q/r^2)\cos\theta = (1/4\pi\varepsilon_0)(2q/r^2)(d/r)$$
$$= (1/4\pi\varepsilon_0)\{2qd/[d^2 + (\ell/2)^2]^{3/2}\}.$$

From the symmetry in the yz-plane, at a point d from the origin we have

$$E = \boxed{(1/4\pi\varepsilon_0)\{2qd/[d^2 + (\ell/2)^2]^{3/2}\}\ \text{away from the origin}}.$$

8. (a) The electric fields produced by the charges will have the same magnitude and point in opposite directions. The resultant field will be $\vec{E} = \boxed{0}$.
(b) Because there is no field at $x = 0$, there will be no force on the test charge. If we displace the test charge a small distance δ away from the x-axis, the two charges will produce a resultant field that will point away from the origin. This is similar to the situation shown in the diagram for Problem 7. Thus there will be a force on the test charge, $\vec{F} = q_0\vec{E}$, that will point away from the origin. The equilibrium will be $\boxed{\text{unstable}}$.

9.



From the diagram, we see that the resultant electric field is

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left[r + (L/2)\right]^2}\hat{i} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\left[r - (L/2)\right]^2}\hat{i}$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{\left[r + (L/2)\right]^2} - \frac{1}{\left[r - (L/2)\right]^2}\right\}\hat{i}$$

$$= \frac{q}{4\pi}\left\{\frac{\left[r - (L/2)\right]^2 - \left[r + (L/2)\right]^2}{\left[r + (L/2)\right]^2\left[r - (L/2)\right]^2}\right\}\hat{i}$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{-2rL}{\left[r + (L/2)\right]^2\left[r - (L/2)\right]^2}\right\}\hat{i}$$

$$= -\frac{2qL}{4\pi\varepsilon_0 r^3}\left\{\frac{1}{\left[1 + (L/2r)\right]^2\left[1 - (L/2r)\right]^2}\right\}\hat{i}.$$

We express this in terms of the dipole moment:

$$\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3}\left\{\frac{1}{\left[1 + (L/2r)\right]^2\left[1 - (L/2r)\right]^2}\right\}.$$

When $r >> L$, the electric field along the axis of the dipole far from the dipole becomes

$$\boxed{\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3}}.$$

10. We treat the line of charges as n pairs symmetrically placed about the y-axis. From the diagram, we see that a pair of charges produces an electric field parallel to the x-axis. For a pair with $r^2 = Y^2 + x^2$, we add the x-components to get the magnitude of the field:

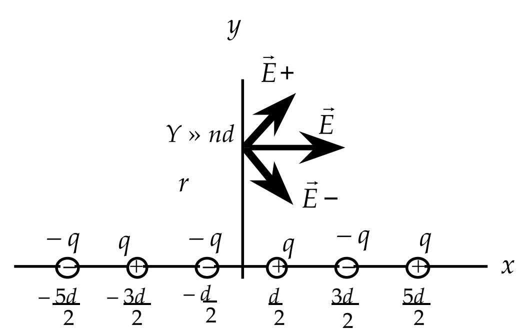
$$E = 2(1/4\pi\varepsilon_0)(q/r^2)(x/r) = 2qx/4\pi\varepsilon_0(Y^2 + x^2)^{3/2}.$$

For all pairs, we have $Y >> x$, so we get

$$E \simeq 2qx/4\pi\varepsilon_0 Y^3.$$

Because the pairs alternate in sign, the direction of E will alternate. The electric field of the ith pair is

$$\vec{E}_i = [(-1)^i 2qx_i/4\pi\varepsilon_0 Y^3]\hat{i}, \text{ with } i = 1, 2, 3, \ldots, n.$$



The values of x_i are $d/2$, $3d/2$, $5d/2$, …, so when we sum the n pairs, we get

$$\vec{E} = \sum\vec{E}_i = \sum[(-1)^i 2qx_i/4\pi\varepsilon_0 Y^3]\hat{i} = (2q/4\pi\varepsilon_0 Y^3)(d/2)(-1 + 3 - 5 + 7 - \ldots)\hat{i}.$$

For the first few terms, the result of the summation is $-1, +2, -3, +4, \ldots$. Thus the general result of the summation is $(-1)^n n$. The resultant electric field is

$$\vec{E} = (2q/4\pi\varepsilon_0 Y^3)(d/2)(-1)^n n\hat{i} = (-1)^n(qnd/4\pi\varepsilon_0 Y^3)\hat{i}.$$

11. We assume that the charge is displaced a small distance δ toward positive x. The net electric field at that point is

$$\vec{E} = (1/4\pi\varepsilon_0)[Q/(a-\delta)^2](-\hat{i}) + (1/4\pi\varepsilon_0)[Q/(a+\delta)^2]\hat{i}$$
$$= (Q/4\pi\varepsilon_0)\{-[1/(a-\delta)^2] + [1/(a+\delta)^2]\}\hat{i}\,.$$

With $\delta << a$, we use the approximation $1/(a+\delta)^2 \simeq (1/a^2) - (2\delta/a^3)$:

$$\vec{E} \simeq (Q/4\pi\varepsilon_0)(-1/a^2 - 2\delta/a^3 + 1/a^2 - 2\delta/a^3)\,\hat{i} = -(4Q\delta/4\pi\varepsilon_0 a^3)\,\hat{i}\,.$$

The force on the test charge is

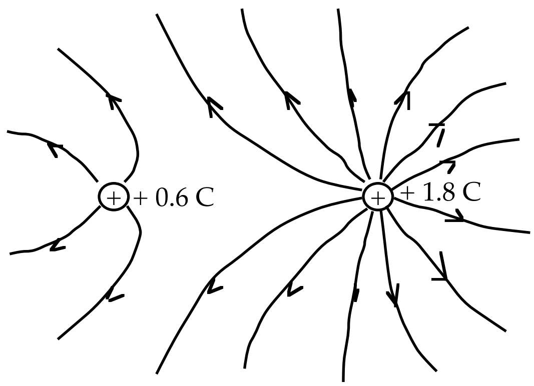
$$\vec{F} = q_0\vec{E} = -(4Q\delta/4\pi\varepsilon_0 a^3)\,\hat{i} = -(Q/\pi\varepsilon_0 a^3)\delta\,\hat{i}\,.$$

Thus the force is a restoring force, so the equilibrium is $\boxed{\text{stable}}$.
The effective force constant of the system is

$k_{\text{eff}} = Q/\pi\varepsilon_0 a^3$, so the frequency is

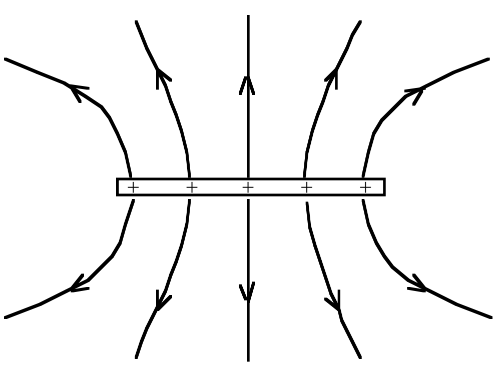
$$f = (1/2\pi)(k_{\text{eff}}/m)^{1/2} = \boxed{(1/2\pi)(Qq/\pi\varepsilon_0 a^3 m)^{1/2}}.$$

12.



13. The density of field lines represents the magnitude of the electric field. Because the electric field between parallel plates depends linearly on the charge density on the plates, the density of the field lines should be $\boxed{\text{tripled}}$.

14.



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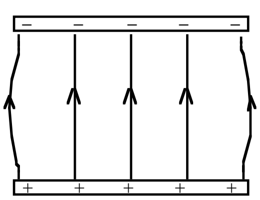

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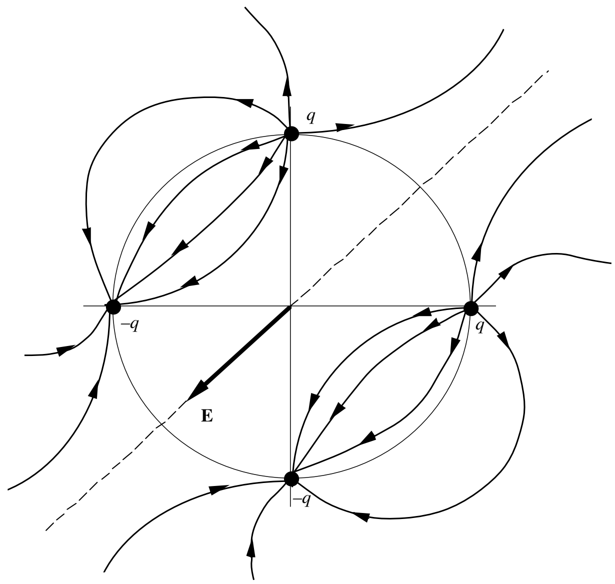
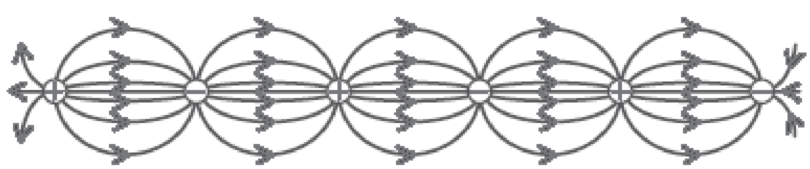


The combined electric field at the center of the turntable due to the two charges at 3- and 9-o'clock is $2kq/R^2$, pointing from the 3 o'clock position to the 9 o'clock position; while that due to the two charges at 12- and 6-o'clock is also $2kq/R^2$ in magnitude, pointing from the 12-o'clock position to the 6-o'clock position. By symmetry the net electric field is

$$E = \sqrt{2}\,(2kq/R^2)$$
$$= 2\sqrt{2}\,(9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(8 \times 10^{-5}\ \mathrm{C})/(0.15\ \mathrm{m})^2 = \boxed{9.1 \times 10^7\ \mathrm{N/C}},$$

pointing midway between the 6- and 9-o'clock positions (i.e., toward the 7:30 position).

17.



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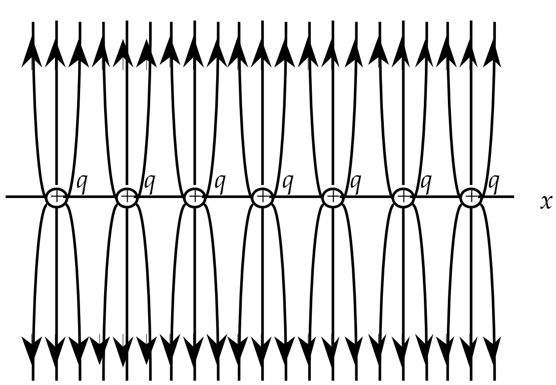
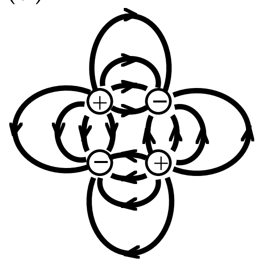
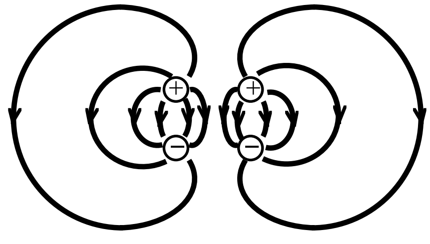


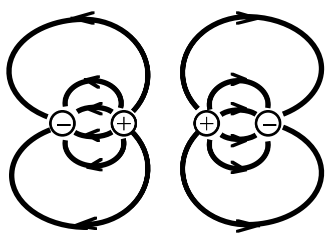
19. (a)



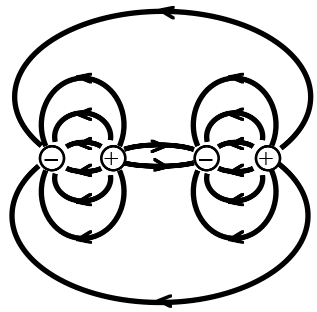
(b)



(c)



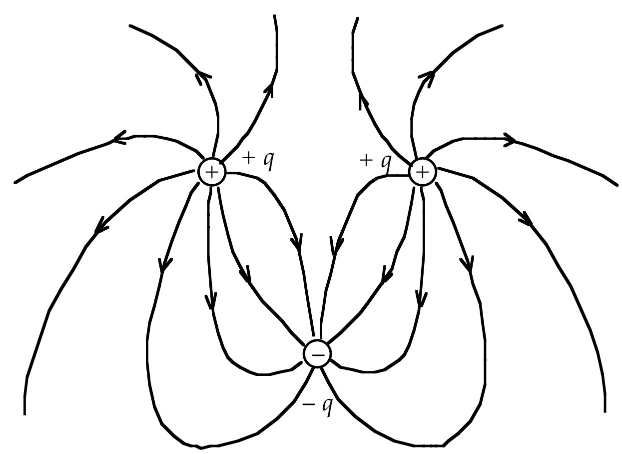
(d)



20. From the dependence of the field on $1/r^2$, close to any single charge that charge will be the major contributor to the field. At a distance of $d = 0.080$ cm from $-q$, the electric field is dominated by that charge. So we have

$\vec{E} \approx kq/d = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})q/(8.0 \times 10^{-4}\ \mathrm{m})^2$

$= \boxed{(1.4 \times 10^{16}\ \mathrm{N/C^2})q \text{ toward } -q}$.



Now consider the field a distance $d_3 = 35$ m from $-q$. Taking the $-q$ position as the origin, east as $+x$ and north as $+y$, the positions of the two $+q$'s are:

$q_1\,(-0.060\ \mathrm{m}, -0.10\ \mathrm{m})$, and $q_2\,(0.06\ \mathrm{m}, -0.104\ \mathrm{m})$.

Therefore, the distance between q_1 and (0, 35 m) is

$d_1 = [(0.060\ \mathrm{m})^2 + (35.10\ \mathrm{m})^2]^{1/2} = 35.1$ m, the same as that between q_2 and (0, 35 m). We have

$E_x = 0$ (by symmetry) and

$E_y = E_{1y} + E_{2y} + E_3 \approx 2kq/d_1^2 - kq/d_3^2$

$= (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})q[2/(35.1\ \mathrm{m})^2 - 1/(35\ \mathrm{m})^2] = (7.3 \times 10^6\ \mathrm{N/C^2})q$. Thus

$\vec{E} = \boxed{[(7.3 \times 10^6\ \mathrm{N/C^2})q]\hat{j}}$ (away from $-q$).

Note that this result can also be obtained by treating the system as one single net charge of $+q$, located 35 m from the point of interest. This is a good approximation since the size of the triangle is much less than 35 m.

21. We find the magnitude of the electric field from an infinitely long line of charge from

$E = \lambda/2\pi\varepsilon_0 R$

$= (0.3 \times 10^{-6}\ \mathrm{C/m})(2)(9 \times 10^9\ \mathrm{N \cdot m^2/C^2})/(20 \times 10^{-2}\ \mathrm{m})$

$= 2.7 \times 10^4\ \mathrm{N/C}$.

The electric field is

$\vec{E} = \boxed{2.7 \times 10^4\ \mathrm{N/C} \text{ perpendicular to and away from the line}}$.

22. The linear charge density is

$$\lambda = Q/L = (6\times10^{-6}\ \mathrm{C})/(0.25\ \mathrm{m}) = 2.4\times10^{-5}\ \mathrm{C/m}.$$

On the x-axis (6 cm, 0 cm, 0 cm) the electric field is

$$dE_x = (k\lambda dz/z^2 + x^2)[x/(z^2 + x^2)^{1/2}] = k\lambda\, x\, dz/(z^2 + x^2)^{3/2}$$

$$E_x = 2k\lambda x \int dz/(z^2 + x^2)^{3/2} \quad \text{(where } z \text{ starts at 0, ends at } 0.25/2\ \mathrm{m)}$$

$$= 2k\lambda\,(\sin\theta_f - \sin\theta_i)/x \quad \text{(where } \theta_i = \text{at } 0^\circ \text{ and } \theta_f = 64.4^\circ)$$

$$= 2(9\times10^9\ \mathrm{N\cdot m^2/C^2})\ (2.4\times10^{-5}\ \mathrm{C/m})(\sin 64.4^\circ - 0)/0.06\ \mathrm{m}$$

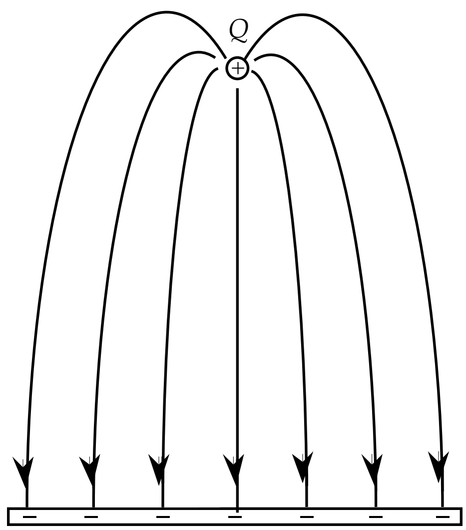
$$= (6.5\times10^6\ \mathrm{N/C}),\ \text{so}$$

$$\vec{E}_1 = E_x\,\hat{i} = \boxed{(6.5\times10^6\ \mathrm{N/C})\hat{i}}.$$

At the same distance from the rod along the y-axis, at (0 cm, 6 cm, 0 cm), the electric field will have the same magnitude but will be in the y-direction:

$$\vec{E}_2 = \boxed{(6.5\times10^6\ \mathrm{N/C})\hat{j}}.$$

23.



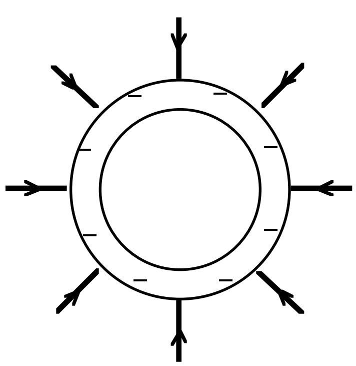
24. Each infinite plane sheet produces a uniform electric field. If we assume both charge densities are positive, between the sheets the fields will be in opposite directions:

$$E = E_1 - E_2 = (\sigma_1 - \sigma_2)/2\varepsilon_0 \text{ away from the first sheet, independent of } L.$$

For the force on charge Q we have

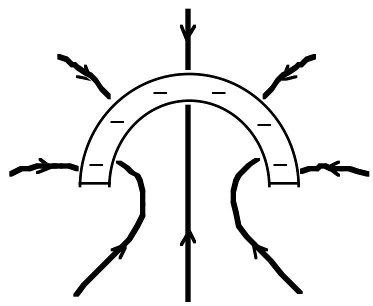
$$F = QE = \boxed{Q(\sigma_1 - \sigma_2)/2\varepsilon_0 \text{ away from the first sheet}}.$$

25.



26. (a) At a point in the plane of the circle outside the circle, the charge distribution will appear as a line. There can be no preference toward either side of the line, so the direction of the electric field must be in the plane of the circle. If the point is moved around the circle, the charge distribution does not change, so the electric field is directed radially out from the center.
(b) From symmetry, the field along the axis of the circle is directed along the axis. At a distance L, with $L >> R$, the charge appears to be a point charge, so we have
$E_{\text{axis}} = (1/4\pi\varepsilon_0)(Q/L^2) = (1/4\pi\varepsilon_0)(2\pi R\lambda/L^2) = \boxed{(R\lambda/2\varepsilon_0 L^2),\ L >> R}$.

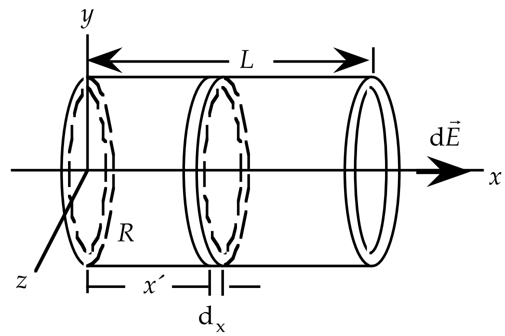
27.



28. Each plate produces an electric field parallel to the x-axis and away from the plate with a magnitude $E = \sigma/2\varepsilon_0$.
(a) Between the plates, the fields from the two plates are in opposite directions, so we have
$E_{0,0,0} = (\sigma/2\varepsilon_0) - (\sigma/2\varepsilon_0) = \boxed{0}$.
(b) Outside the two plates, the fields from the two plates are in the same direction, so we have
$\vec{E}_{8,0,0} = (\sigma/2\varepsilon_0)\,\hat{i} + (\sigma/2\varepsilon_0)\,\hat{i} = (\sigma/\varepsilon_0)\,\hat{i}$
$= [(1.2 \times 10^{-6}\ \text{C/m}^2)/(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)]\,\hat{i}$
$= \boxed{(1.4 \times 10^5\ \text{N/C})\hat{i}}$.
(c) The field outside the plates is independent of y and z, so we have
$\vec{E}_{8,1,2} = \boxed{(1.4 \times 10^5\ \text{N/C})\hat{i}}$.

29. We assume that the plates are large enough that they may be considered infinite plates. Each plate produces an electric field perpendicular to and away from the plate with a magnitude
$E = \sigma/2\varepsilon_0$.
Outside the two plates, the fields from the two plates are in the same direction, so we have
$E = (\sigma/2\varepsilon_0) + (\sigma/2\varepsilon_0)$
$= \boxed{\sigma/\varepsilon_0 \text{ perpendicular to the plates and away from them}}$.
Between the plates, the fields from the two plates are in opposite directions, so we have
$E = (\sigma/2\varepsilon_0) - (\sigma/2\varepsilon_0) = \boxed{0}$.

30. Because we know the electric field for a hoop, we choose a circular segment of length dx' as the differential element. The charge on this segment is $dq = (q/L)\,dx'$. The field of this element at a point x on the x-axis will be in the $+x$-direction, with a magnitude

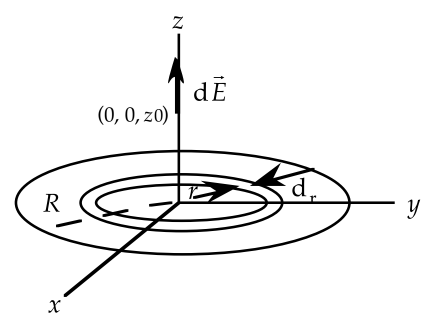


$$dE = \frac{1}{4\pi\varepsilon_0}\frac{(x-x')\,dq}{\left[R^2+(x-x')^2\right]^{3/2}}$$
$$= \frac{q}{4\pi\varepsilon_0 L}\frac{(x-x')\,dx'}{\left[R^2+(x-x')^2\right]^{3/2}}.$$

We find the total field by integrating over the length of the tube:

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 L}\int\frac{(x-x')\,dx'}{\left[R^2+(x-x')^2\right]^{3/2}}\hat{i} = \frac{q}{4\pi\varepsilon_0 L}\left\{\frac{1}{\left[R^2+(x-x')^2\right]^{1/2}}\right\}\hat{i}\,\Bigg|_0^L$$
$$= \frac{q}{4\pi\varepsilon_0 L}\left\{\frac{1}{\left[R^2+(x-L)^2\right]^{1/2}} - \frac{1}{\left[R^2+x^2\right]^{1/2}}\right\}\hat{i}.$$

$\boxed{31}$. (a) From the symmetry of the charge distribution, we know that the electric field on the z-axis is along the z-axis. For a differential element we choose a ring of radius r and thickness dr. The charge on the ring is $dq = (Q/\pi R^2)2\pi r\,dr = (2Qr\,dr)/R^2$. Using the result for the field of a hoop of charge, we integrate over the disk:



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{z_0\,dq}{(r^2+z_0^2)^{3/2}}\hat{k} = \frac{z_0 2Q}{4\pi\varepsilon_0 R^2}\int_0^R\frac{r\,dr}{(r^2+z_0^2)^{3/2}}\hat{k}$$
$$= \frac{z_0 Q}{2\pi\varepsilon_0 R^2}\left[\frac{-1}{(r^2+z_0^2)^{1/2}}\right]\Bigg|_0^R\hat{k}$$
$$= \frac{z_0 Q}{2\pi\varepsilon_0 R^2}\left[\frac{1}{z_0} - \frac{1}{(R^2+z_0^2)^{1/2}}\right]\hat{k}.$$

(b) To find the field in the limit $z_0 \to \infty$, we rearrange and use the approximation $(1+x)^{-1/2} \simeq 1-(x/2)$:

$$\vec{E} = \frac{Q}{2\pi\varepsilon_0 R^2}\left\{1 - \left[1+\left(\frac{R}{z_0}\right)^2\right]^{-1/2}\right\}\hat{k} = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - 1 + \frac{1}{2}\left(\frac{R}{z_0}\right)^2\right]\hat{k}$$
$$= \frac{Q}{4\pi\varepsilon_0 z_0^2}\hat{k}.$$

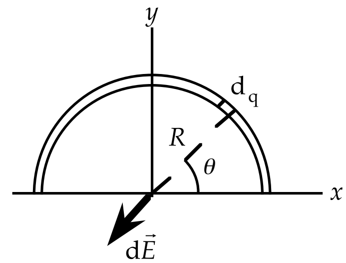
As we expect, the field is that of a point charge.

(c) To find the field in the limit $R \to \infty$, we consider the result from part (a). The second term will go to zero, so we have $\vec{E} = Q/2\pi\varepsilon_0 R^2\,\hat{k}$.

The charge density of the disk is $\sigma = Q/\pi R^2$, so we can write $\vec{E} = \boxed{(\sigma/2\varepsilon_0)\hat{k}}$.
As we expect, the field is that of an infinite plane.
The limits of parts (b) and (c) are not the same. Part (b) is equivalent to the disk being a point charge, while part (c) is equivalent to being very close to the disk.

32. Let the length of the rod be L, then the radius of the semicircle is $R = L/\pi$. Because the charge distribution is symmetric about the y-axis, we know that the electric field at the center will be directed along the $-y$-axis. We choose a differential element of the rod at an angle θ with charge

$$dq = Q\,(d\theta/\pi).$$



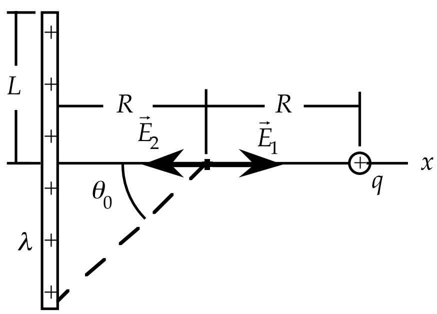
We find the total field at the center by integrating the y-components over the rod:

$$dE_y = -(k\,dq\,\sin\theta/R^2) = -k[(Q/\pi)\,d\theta]\,\sin\theta/(L/\pi)^2$$
$$= -(\pi kQ/L^2)\,\sin\theta\,d\theta\,;$$
$$E_y = -(\pi kQ/L^2)\int \sin\theta\,d\theta\,,$$ where θ starts at 0° and ends at 180°.

Thus

$$\vec{E} = E_y\,\hat{j} = (-2\pi kQ/L^2)\,\hat{j} = [-2\pi\,(9\times10^9\ \mathrm{N\cdot m^2/C^2})(0.36\times10^{-6}\ \mathrm{C})/(0.18\ \mathrm{m})^2]\,\hat{j}$$
$$= \boxed{-(6.3\times10^5\ \mathrm{N/C})\hat{j}}.$$

33.



We find the electric field from the vector sum of the field of a rod and the field of a point charge:

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$
$$= (\lambda/2\pi\varepsilon_0 R)\,\sin\theta_0\,\hat{i} - (q/4\pi\varepsilon_0 R^2)\,\hat{i}$$
$$= (1/4\pi\varepsilon_0)[(2\lambda\sin\theta_0)/R - q/R^2]\,\hat{i}\,.$$

The angle for the endpoint of the rod is

$$\theta_0 = \tan^{-1}(L/R) = \tan^{-1}(1) = 45°.$$

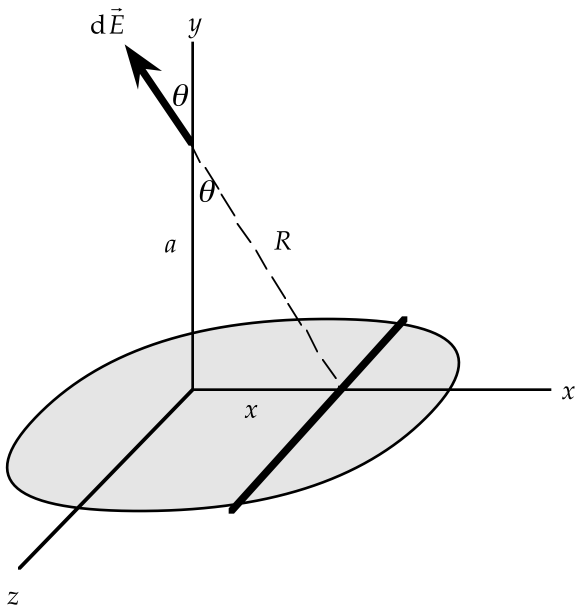
The magnitude of the field is

$$E = (9\times10^9\ \mathrm{N\cdot m^2/C^2})[2(15\times10^{-6}\ \mathrm{C/m})(\sin 45°)/(0.15\ \mathrm{m}) - (3\times10^{-6}\ \mathrm{C})/(0.15\ \mathrm{m})^2]$$
$$= 7.3\times10^4\ \mathrm{N/C}.$$

The resultant field is

$$\vec{E} = \boxed{7.3\times10^4\ \mathrm{N/C}\ \ \text{toward the point charge}}.$$

34.



Imagine that the infinitely large sheet is made of infinitely long rods running in the z-direction in the xz plane. Consider one such rod, of width dx, a distance x from the z-axis. The charge per unit length of the rod is $\lambda = dq/(\text{length of the rod}) = \sigma\, dx$. According to the result of Example 22-7, with the length of the rod approaching infinity, the magnitude of the electric field dE due to the rod at a point on the y-axis a distance a from the charged sheet is

$$dE = \lambda/2\pi\varepsilon_0 R = \sigma\, dx/2\pi\varepsilon_0 R.$$

By symmetry, the net field at this point is along the y-axis, so we only need to consider the y-component of $d\vec{E}$:

$$dE_y = dE\cos\theta = (\lambda/2\pi\varepsilon_0 R)(a/R)$$
$$= \sigma a\, dx/2\pi\varepsilon_0 R^2 = \sigma a\, dx/2\pi\varepsilon_0(x^2 + a^2).$$

Integrate over the entire range of x:

$E = \int dE_y = (\sigma a/2\pi\varepsilon_0)\int dx/(x^2 + a^2)$, where x ranges from $-\infty$ to $+\infty$.

The integral in the last step is equal to $(1/a)\tan^{-1}(x/a)$, and with the upper- and lower-limits given above it yields π/a. Thus

$E = (\sigma a/2\pi\varepsilon_0)(\pi/a) = \sigma/2\varepsilon_0$, as expected.

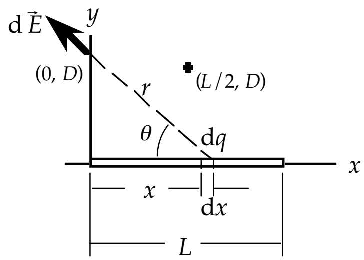
35. Since the electric field at the radius $(R + h)$ vanishes, and the charge distribution within the radius is spherically symmetrical, the total charge enclosed within the radius must be zero. This means that the charge on the shell of thickness h must be $+Q$. Since $R >> h$ the shell is very thin, so its radius is nearly uniform, and is approximately equal to R. The volume of the shell is then

$$V \approx 4\pi R^2 h.$$

The charge density on the shell follows as

$$\rho = Q/V \approx \boxed{Q/4\pi R^2 h}.$$

36.



To find the electric field at the point (0, D), we choose a differential element of the rod, as shown in the diagram. The charge of this element is $dq = (Q/L)\,dx$. We find the field produced by the element, which has both x- and y-components, by integrating along the rod:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{x=0}^{x=L} \frac{dq}{r^2}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\int_{x=0}^{x=L} \frac{dx}{r^2}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right).$$

To perform the integration, we must eliminate variables until we have one, for which we choose θ.
From the diagram we see that $r = D/\sin\theta$, and $x = D\cot\theta$. This gives

$$dx = -D\csc^2\theta\,d\theta = -(D\,d\theta)/\sin^2\theta.$$

The limits for θ are $\pi/2$ rad to $\theta_0 = \cos^{-1}[L/(D^2 + L^2)]$. When we make these substitutions, we have

$$\vec{E}(0, D) = \frac{Q}{4\pi\varepsilon_0 L}\int_{\pi/2}^{\theta_0} \frac{(-d\theta)/\sin^2\theta}{(D/\sin\theta)^2}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 LD}\int_{\pi/2}^{\theta_0} d\theta\left(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 LD}\left(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)\Big|_{\pi/2}^{\theta_0}$$

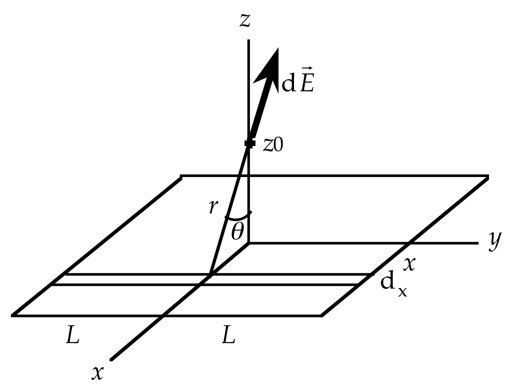
$$= \frac{Q}{4\pi\varepsilon_0 LD}\left[(\sin\theta_0 - 1)\,\hat{i} + (\cos\theta_0 - 0)\,\hat{j}\right];$$

$$\vec{E}(0, D) = \frac{Q}{4\pi\varepsilon_0 LD}\left[\left(\frac{D}{\sqrt{D^2 + L^2}} - 1\right)\hat{i} + \left(\frac{L}{\sqrt{D^2 + L^2}}\right)\hat{j}\right].$$

Because the point ($L/2$, D) is opposite the midpoint of the rod, we know that the field there will have only a y-component. Instead of doing another integration, we use the result from the text:

$$\vec{E}(L/2, D) = \frac{2\lambda}{4\pi\varepsilon_0 D}\left[\frac{L/2}{\sqrt{D^2 + (L/2)^2}}\right]\hat{j} = \frac{Q}{4\pi\varepsilon_0 D}\left(\frac{2}{\sqrt{4D^2 + L^2}}\right)\hat{j}.$$

37. (a) We choose a strip of the square parallel to the y-axis, so that we can use the result for the electric field of a rod. The strip has length $2L$ and thickness dx. The charge of the strip is d$q = \sigma 2L$ dx, which gives a linear charge density $\lambda = \sigma$ dx.



The magnitude of the field produced by the strip is

$$dE = \frac{\sigma\, dx}{2\pi\varepsilon_0 r}\frac{L}{\sqrt{L^2 + r^2}}.$$

From the diagram, we see that the charge distribution is symmetric about the y-axis. The resultant field will be in the z-direction, so we integrate the z-components for the region from $x = -L$ to $x = L$.

From the diagram, we have $\cos\theta = z_0/r$.

$$\vec{E} = \int_{x=-L}^{x=L} \frac{\sigma\, dx}{2\pi\varepsilon_0 r}\frac{L}{\sqrt{L^2 + r^2}} \cos\theta\, \hat{k} = \frac{\sigma L z_0}{2\pi\varepsilon_0}\int_{x=-L}^{x=L} \frac{dx}{r^2 L^2 + r^2}\, \hat{k}$$

$$= \frac{\sigma L z_0}{2\pi\varepsilon_0}\, \hat{k} \int_{x=-L}^{x=L} \frac{dx}{(x^2 + z_0^2)\sqrt{L^2 + x^2 + z_0^2}}.$$

(b) We can simplify the integral by doubling the z-components for the region from $x = 0$ to $x = L$:

$$\vec{E} = 2\int_{x=0}^{x=L} \frac{\sigma\, dx}{2\pi\varepsilon_0 r}\frac{L}{\sqrt{L^2 + r^2}} \cos\theta\ \hat{k}.$$

We choose θ for the variable by using $x = z_0 \tan\theta$, $dx = (z_0/\cos^2\theta)\, d\theta$, $r = z_0/\cos\theta$.

The limits for θ are 0 and θ_0, which is determined from $\tan\theta_0 = L/z_0$.

The integral becomes

$$\vec{E} = \frac{2\sigma}{2\pi\varepsilon_0}\int_0^{\theta_0} \frac{(z_0/\cos^2\theta)\, d\theta}{(z_0/\cos\theta)} \frac{L}{\sqrt{L^2 + (z_0/\cos\theta)^2}} \cos\theta\ \hat{k}$$

$$= \frac{\sigma}{\pi\varepsilon_0}\int_0^{\theta_0} \frac{\cos\theta\, d\theta}{\sqrt{\cos^2\theta + (z_0/L)^2}}\ \hat{k}.$$

As $L \to \infty$, the denominator in the integrand becomes $\cos\theta$, so the integral is $\int d\theta$. We find the upper limit of the integral from $\tan\theta_0 \to \infty$, which gives $\theta_0 = \pi/2$. The value of the integral is $\pi/2$, and the field becomes

$$\vec{E} \to (\sigma/\pi\varepsilon_0)(\pi/2)\hat{k} = \boxed{(\sigma/2\varepsilon_0)\hat{k}}.$$

(c) As $z_0 \to 0$, the denominator in the integrand becomes $\cos\theta$, so the integral is $\int d\theta$. We find the upper limit of the integral from $\tan\theta_0 \to \infty$, which gives $\theta_0 = \pi/2$. The value of the integral is $\pi/2$, and the field becomes

$$\vec{E} \to (\sigma/\pi\varepsilon_0)(\pi/2)\hat{k} = \boxed{(\sigma/2\varepsilon_0)\hat{k}}.$$

This is the result from part (b). In both cases, the square looks like an infinite plane.

38. Because the electric field produced by the infinite plate is constant, there will be a constant downward force on the charge and thus constant acceleration of the pellet:

$$a = |q|E/m = |q|\sigma/2\varepsilon_0 m.$$

We find the speed from

$$v^2 = v_0^2 + 2a(y - y_0) = 0 + 2(|q|\sigma/2\varepsilon_0 m)(d - 0)$$

$$= [(1.08 \times 10^{-6}\ \text{C})(2.17 \times 10^{-6}\ \text{C/m}^2)/(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.555 \times 10^{-3}\ \text{kg})](0.175\ \text{m}),$$

which gives $\boxed{v = 9.1\ \text{m/s}}$.

39. The force from the electric field produces the acceleration:

$qE = ma;$

$q(850\ \mathrm{N/C}) = (120 \times 10^{-6}\ \mathrm{kg})(4.6\ \mathrm{m/s^2})$, which gives $q = 6.5 \times 10^{-7}\ \mathrm{C} = \boxed{0.65\ \mu\mathrm{C}}$.

40. The uniform electric field from the sheet produces a force on the electron toward the sheet which gives the electron an acceleration:

$qE = q\sigma/2\varepsilon_0 = ma$, or $a = q\sigma/2\varepsilon_0 m$.

We find the velocity from

$v = v_0 + at = 0 + (q\sigma/2\varepsilon_0 m)t$

$= (1.6 \times 10^{-19}\ \mathrm{C})(6.1 \times 10^{-9}\ \mathrm{C/m^2})/[2(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(9.1 \times 10^{-31}\ \mathrm{kg})](17.5 \times 10^{-9}\ \mathrm{s})$

$= \boxed{1.1 \times 10^6\ \mathrm{m/s}}$ away from the sheet.

We check the distance traveled by the electron:

$d = \frac{1}{2}(v + v_0)t = \frac{1}{2}(1.01 \times 10^6\ \mathrm{m/s})(17.5 \times 10^{-9}\ \mathrm{s}) = \boxed{9.6 \times 10^{-3}\ \mathrm{m}}$.

41. Because the electric field produced by the infinite plate is constant, there must be a constant upward force on the charge that balances the downward force of gravity. To produce an upward force on a positive charge, the plate must have a positive charge. We find the density from

$qE = q(\sigma/2\varepsilon_0) = mg;$

$(8.5 \times 10^{-7}\ \mathrm{C})\sigma/2(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}) = (0.83 \times 10^{-3}\ \mathrm{kg})(9.8\ \mathrm{m/s^2})$, which gives

$\sigma = \boxed{1.7 \times 10^{-7}\ \mathrm{C/m^2}}$.

42. Because we can treat the nucleus as a point charge, the field will be radial:

$E = [(1/4\pi\varepsilon_0)(q/r^2)]$

$= [(9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(79)(1.60 \times 10^{-19}\ \mathrm{C})/(10^{-11}\ \mathrm{m})^2] = \boxed{(1.14 \times 10^{15}\ \mathrm{N/C})\ \text{radial}}$.

The acceleration of the alpha particle is

$a = QE/m = 2(1.60 \times 10^{-19}\ \mathrm{C})(1.14 \times 10^{15}\ \mathrm{N/C})/[4(1.67 \times 10^{-27}\ \mathrm{kg})]$

$= \boxed{5.4 \times 10^{22}\ \mathrm{m/s^2}\ \text{(away from the nucleus)}}$.

43. The force produced by the electric field of the wire on the negative charge is toward the wire and provides the centripetal force:

$F = mv^2/r;$

$q(\lambda/2\pi\varepsilon_0 r) = mv^2/r$, which gives a speed $v = \boxed{(q\lambda/2\pi\varepsilon_0 m)^{1/2}}$, which does not depend on r.

44. The force produced by the electric field of the wire on the negative charge is toward the wire. We choose the x-axis along the wire and the y-axis perpendicular to the wire to apply $\sum \vec{F} = m\vec{a}$:

x-component: $0 = m\,\mathrm{d}^2x/\mathrm{d}t^2$, which we normally write as $\boxed{m\,\mathrm{d}^2x/\mathrm{d}t^2 = 0}$;

y-component: $-q\lambda/2\pi\varepsilon_0 y = m\,\mathrm{d}^2y/\mathrm{d}t^2$, which we normally write as $\boxed{m\,\mathrm{d}^2y/\mathrm{d}t^2 = -q\lambda/2\pi\varepsilon_0 y}$.

45. The force produced by the electric field of the wire on the negative charge is toward the wire and provides the centripetal force:

$F = mv^2/r;$

$q(\lambda/2\pi\varepsilon_0 r) = mv^2/r$, which gives a speed $v = (q\lambda/2\pi\varepsilon_0 m)^{1/2}$.

The period of the orbit is

$T = 2\pi r/v = [2\pi/(q\lambda/2\pi\varepsilon_0 m)^{1/2}]r$.

If the centripetal force is provided by a point charge, we have

$(1/4\pi\varepsilon_0)(qQ/r^2) = mv^2/r$, which gives

$v = (qQ/4\pi\varepsilon_0 mr)^{1/2}$.

The period of the orbit is

$T_{\text{point charge}} = 2\pi r/v = 2\pi(4\pi\varepsilon_0 m/qQ)^{1/2} r^{3/2}$, which has a different r dependence.

46. The electric field of each plate is up, and the electric force must be up to balance the force of gravity; therefore the charge must be positive. Because the acceleration is zero, we have

$$qE_+ + qE_- = mg;$$
$$q = mg/[(\sigma_+/2\varepsilon_0) + (\sigma_2/2\varepsilon_0)] = 2mg\varepsilon_0/(\sigma_+ + \sigma_-)$$
$$= 2(5.6\times 10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)/[(1.6 + 0.22)\times 10^{-6}\ \text{C/m}^2] = \boxed{5.3\times 10^{-7}\ \text{C}}.$$

47. We use the coordinate system from Example 22–12. The initial horizontal component of the velocity is

$$v_{0x} = (v_0^2 - v_{0y}^2)^{1/2} = [(5.0\times 10^6\ \text{m/s})^2 - (2.0\times 10^5\ \text{m/s})^2]^{1/2} = 5.0\times 10^6\ \text{m/s}.$$

The time for the electron to travel between the plates is

$$t_1 = L_1/v_{0x} = (3\times 10^{-2}\ \text{m})/(5.0\times 10^6\ \text{m/s}) = 6.0\times 10^{-9}\ \text{s}.$$

The deflection at this time is

$$y_1 = v_{0y}t_1 + \tfrac{1}{2}at_1^2 = v_{0y}t_1 + \tfrac{1}{2}(qE/m)t_1^2$$
$$= (2.0\times 10^5\ \text{m/s})(6.0\times 10^{-9}\ \text{s}) + \tfrac{1}{2}[(1.6\times 10^{-19}\ \text{C})(10^3\ \text{N/C})/(9.1\times 10^{-31}\ \text{kg})](6.0\times 10^{-9}\ \text{s})^2$$
$$= 4.4\times 10^{-3}\ \text{m}.$$

The vertical component of the velocity as the electron leaves the plates is

$$v_{1y} = v_{0y} + at_1 = v_{0y} + (qE/m)t_1$$
$$= (2.0\times 10^5\ \text{m/s}) + [(1.6\times 10^{-19}\ \text{C})(10^3\ \text{N/C})/(9.1\times 10^{-31}\ \text{kg})](6.0\times 10^{-9}\ \text{s})$$
$$= 1.25\times 10^6\ \text{m/s}.$$

After it leaves the plates, the electron travels in a straight line with a direction given by

$$\tan\theta = v_{1y}/v_{0x} = (1.25\times 10^6\ \text{m/s})/(5.0\times 10^6\ \text{m/s}) = 0.25.$$

The deflection while the electron travels this straight line is

$$y_2 = L_2\tan\theta = (12\times 10^{-2}\ \text{m})(0.25) = 3.0\times 10^{-2}\ \text{m}.$$

The total deflection is

$$y = y_1 + y_2 = (0.44\times 10^{-2}\ \text{m}) + (3.0\times 10^{-2}\ \text{m}) = 3.4\times 10^{-2}\ \text{m} = \boxed{3.4\ \text{cm}}.$$

48. The electric field between the oppositely-charged parallel plates is uniform and will produce a constant acceleration:

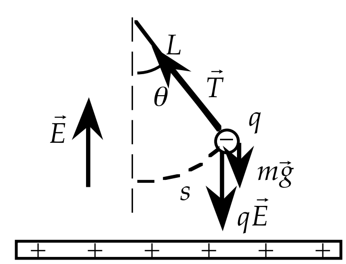
$$qE = e\sigma/\varepsilon_0 = ma, \text{ or } a = e\sigma/\varepsilon_0 m.$$

For a constant acceleration over the separation of the plates d, we have

$$v^2 = v_0^2 + 2ad;$$
$$(3.0\times 10^7\ \text{m/s})^2 = (1.6\times 10^6\ \text{m/s})^2 + 2[(1.6\times 10^{-19}\ \text{C})\sigma/(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(9.1\times 10^{-31}\ \text{kg})](2\times 10^{-2}\ \text{m}),$$

which gives $\sigma = \boxed{1.1\times 10^{-6}\ \text{C/m}^2}$.

49. The electric field produced by the plate is



$$E = \sigma/2\varepsilon_0$$
$$= (10^{-6}\ \text{C/m}^2)/2(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 5.6\times 10^4\ \text{N/C}.$$

Using the force diagram, we find the equation of motion for the tangential direction:

$$-(mg + qE)\sin\theta = m\,d^2s/dt^2.$$

If the angle is small, we have

$\sin\theta \simeq \theta = s/L$, and the equation of motion becomes

$$-[(mg + qE)/L]s = m\,d^2s/dt^2.$$

This is the equation for simple harmonic motion. The effective force constant is

$$k_{\text{eff}} = (mg + qE)/L.$$

The angular frequency of the motion is

$$\omega = (k_{\text{eff}}/m)^{1/2} = [(mg + qE)/Lm]^{1/2}$$
$$= \{[(5\times 10^{-3}\ \text{kg})(9.8\ \text{m/s}^2) + (2\times 10^{-6}\ \text{C})(5.6\times 10^4\ \text{N/C})]/(1\ \text{m})(5\times 10^{-3}\ \text{kg})\}^{1/2} = \boxed{5.7\ \text{s}^{-1}}.$$

50. To find the force on the proton, we need the electric field. We write the linear dependence on x and find the constants:

$E = E_0 - bx$;

$500\text{ N/C} = E_0 - 0$, which gives $E_0 = 500\text{ N/C}$;

$0 = 500\text{ N/C} - b(3\text{ m})$, which gives $b = (500/3)\text{ N/C}\cdot\text{m}$.

Because the electric force is the only force on the proton, the equation of motion is

$m\,d^2x/dt^2 = q(E_0 - bx)$.

If we change variable to $x' = x - 3$, $d^2x'/dt^2 = d^2x/dt^2$, and we have

$m\,d^2x'/dt^2 = q(E_0 - bx' - 3b) = -qbx'$.

We rewrite this as

$d^2x'/dt^2 = -(qb/m)x' = -\omega^2x'$,

which is the equation for simple harmonic motion, with angular frequency

$\omega = (qb/m)^{1/2} = [(1.60 \times 10^{-19}\text{ C})(167\text{ N/C}\cdot\text{m})/(1.67 \times 10^{-27}\text{ kg})]^{1/2} = 1.26 \times 10^5\text{ s}^{-1}$.

We write the solution:

$x'(t) = A \sin(\omega t + \delta)$, and $v(t) = A\omega \cos(\omega t + \delta)$,

and determine A and δ from the initial conditions:

$x'(0) = -3\text{ m} = A \sin(0 + \delta)$; $v(0) = 5 \times 10^5\text{ m/s} = -A\omega \cos(0 + \delta)$.

The solution of these two equations is

$A = 5.0\text{ m}$ and $\delta = -37°$.

The time to traverse the region is the time when $x = 3$ m, or $x' = 0$:

$0 = (5.0\text{ m}) \sin(\omega t - 37°)$, which gives $\omega t = (37°)(\pi/180°)$, or

$t = (37°)(\pi/180°)/(1.26 \times 10^5\text{ s}^{-1}) = \boxed{5.1 \times 10^{-6}\text{ s}}$.

51. The torque on the dipole is

$$\begin{aligned}\vec{\tau} = \vec{p} \times \vec{E} &= qL(\cos 45°\,\hat{i} + \sin 45°\,\hat{j}) \times E\hat{i}\\ &= -qLE \sin 45°\,\hat{k}\\ &= (2 \times 10^{-6}\text{ C})(0.10\text{ m})(10\text{ N/C}) \sin 45°\,\hat{k}\\ &= \boxed{-(1.41 \times 10^{-6}\text{ N}\cdot\text{m})\hat{k}}.\end{aligned}$$

52. The torque is directly proportional to the magnitude of the dipole moment. The new dipole moment is

$p_2 = q_2L_2 = (5q_1)(3L_1) = 15q_1L_1 = \boxed{15p_1}$.

The torque will be increased by a factor of 15.

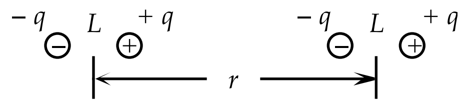
$\boxed{53}$. We estimate the field along the bisector:

$$\begin{aligned}E &\simeq (1/4\pi\varepsilon_0)(p/r)^3\\ &= [(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6 \times 10^{-30}\text{ C}\cdot\text{m})/(3 \times 10^{-9}\text{ m})^3]\\ &= \boxed{2 \times 10^6\text{ N/C}}.\end{aligned}$$

54. From Problem 51, the torque acts to align the dipole with the electric field. As the dipole passes the x-axis, the torque direction will reverse. The dipole will oscillate around the direction of the electric field. The work done by the electric field is

$$\begin{aligned}W &= -\Delta U = -[(-\vec{p} \cdot \vec{E})_f - (-\vec{p} \cdot \vec{E})_i]\\ &= -[(-pE) - (-pE \cos 45°)]\\ &= qLE(1 - \cos 45°)\\ &= (2 \times 10^{-6}\text{ C})(0.10\text{ m})(10\text{ N/C})(1 - 0.707)\\ &= \boxed{5.9 \times 10^{-7}\text{ J}}.\end{aligned}$$

55. Each dipole is a pair of charges q separated by a distance L, such that $p = qL$. To find the force on the dipole on the right, we find the electric field at each of the charges produced by the charges of the other dipole. The field at the positive charge is



$$E_+ = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r^2} - \frac{q}{(r+L)^2}\right] = \frac{q}{4\pi\varepsilon_0 r^2}\left\{1 - \frac{1}{\left[1+(L/r)\right]^2}\right\} \text{ to the right.}$$

The field at the negative charge is

$$E_- = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-L)^2} - \frac{q}{r^2}\right] = \frac{q}{4\pi\varepsilon_0 r^2}\left\{\frac{1}{\left[1-(L/r)\right]^2} - 1\right\} \text{ to the right.}$$

The force on the dipole is

$$F = qE_+ + (-q)E_- = \frac{q^2}{4\pi\varepsilon_0 r^2}\left\{1 - \frac{1}{\left[1+(L/r)\right]^2} 1 - \frac{1}{\left[1-(L/r)\right]^2} + 1\right\} \text{ to the right.}$$

Because $L << r$, we make use of the approximation $(1 \pm x)^{-2} \simeq 1 \mp 2x + 3x^2 \mp \ldots$ and expand the terms:

$$F \simeq \frac{q^2}{4\pi\varepsilon_0 r^2}\left\{1 - \left[1 - 2\frac{L}{r} + 3\left(\frac{L}{r}\right)^2\right] - \left[1 + 2\frac{L}{r} + 3\left(\frac{L}{r}\right)^2\right] + 1\right\} = \frac{q^2}{4\pi\varepsilon_0 r^2}\left[-6\left(\frac{L}{r}\right)^2\right] = -\frac{6q^2L^2}{4\pi\varepsilon_0 r^4} = -\frac{6p^2}{4\pi\varepsilon_0 r^4} \text{(attraction).}$$

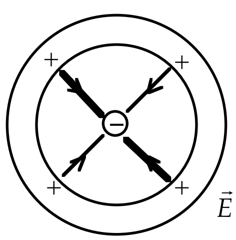
56. The potential energy of a dipole in an electric field is $U = -\vec{p} \cdot \vec{E}$. The maximum energy occurs when $\vec{p}$ and $\vec{E}$ are in opposite directions, and the minimum energy occurs when $\vec{p}$ and $\vec{E}$ are parallel:

$U_{\text{max}} = pE$, $U_{\text{min}} = -pE$, and

$\Delta U = 2pE$;

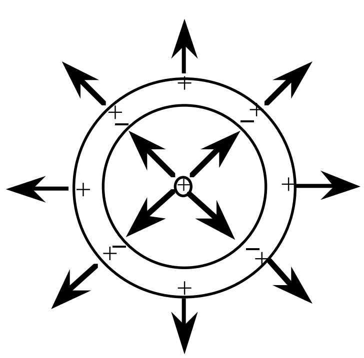
4.4×10^{-25} J $= 2p(10^4$ N/C), which gives $p = \boxed{2.2 \times 10^{-29}\ \text{C}\cdot\text{m}}$.

$\boxed{57}$.

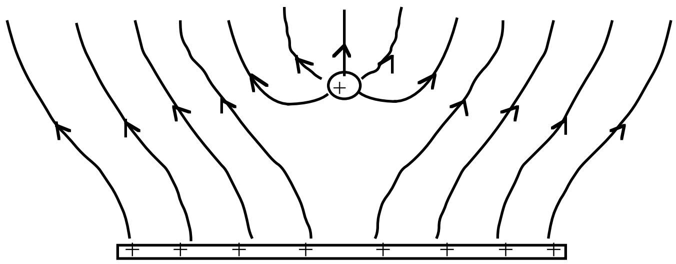


$\vec{E} = 0$ outside

58.

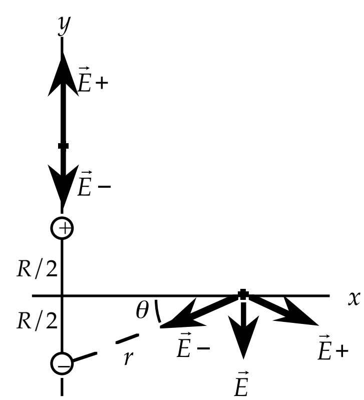


59.



60. In the region outside a uniformly charged sphere, the electric field is the same as that of a point charge:
$E = (1/4\pi\varepsilon_0)(q/r^2)$.
For the cork ball, we have
$E = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(3.5 \times 10^{-9}\ \mathrm{C})/(1.2 \times 10^{-2}\ \mathrm{m})^2 = \boxed{2.2 \times 10^5\ \mathrm{N/C}}$.
For the uranium nucleus, we have
$E = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(28)(1.6 \times 10^{-19}\ \mathrm{C})/(5 \times 10^{-15}\ \mathrm{m})^2 = \boxed{1.6 \times 10^{21}\ \mathrm{N/C}}$.

61. We choose the coordinate system shown in the diagram, with the rods aligned parallel to the z-axis.
(a) $\vec{E} = \vec{E}_+ + \vec{E}_- = (\lambda/2\pi\varepsilon_0)[1/(y - R/2) - 1/(y + R/2)]\,\hat{j}$
$= \boxed{\{(\lambda R/2\pi\varepsilon_0)/[y^2 - (R/2)^2]\}\hat{j}}$.
(b) We see from the diagram that the symmetry along the x-axis means that the resultant field will have only a y-component. We find the field by doubling the y-component from one rod:
$\vec{E} = -2E_- \sin\theta\,\hat{j} = -(2\lambda/2\pi\varepsilon_0 r)[(R/2)/r]\,\hat{j}$
$= -(\lambda R/2\pi\varepsilon_0 r^2)\,\hat{j} = \boxed{-\{(\lambda R/2\pi\varepsilon_0)/[x^2 + (R/2)^2]\}\hat{j}}$.

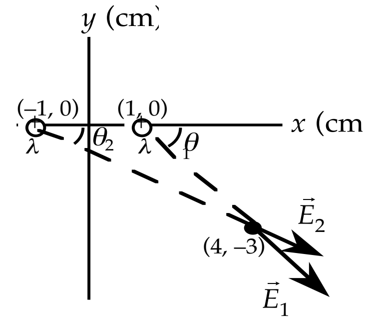


62. The electric field at the positive rod produced by the negative rod is
$E = \lambda/2\pi\varepsilon_0 R$ toward the negative rod.
The force on a charge Q of the rod is $F = QE$, so the force per unit length is
$F/L = (Q/L)E$
$= \lambda(\lambda/2\pi\varepsilon_0 R) = \boxed{\lambda^2/2\pi\varepsilon_0 R\ \text{(attraction)}}$.

63. Each infinite plate produces a constant field perpendicular to the plate. The total electric field is
$\vec{E} = (\sigma_1/2\varepsilon_0)\,\hat{j} + (\sigma_2/2\varepsilon_0)\,\hat{i}$.
The force produced by this field on the particle causes a constant acceleration:
$\vec{a} = q\vec{E}/m = (q/2\varepsilon_0 m)(\sigma_1\hat{j} + \sigma_2\hat{i})$.
If the particle starts from rest, its position is
$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = (1\ \mathrm{m})\hat{i} + (1\ \mathrm{m})\hat{j} + 0 + (q/4\varepsilon_0 m)(\sigma_2\hat{i} + \sigma_1\hat{j})t^2$
$= \{1 + [(1 \times 10^{-7}\ \mathrm{C})/4(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(1 \times 10^{-3}\ \mathrm{kg})](+3 \times 10^{-6}\ \mathrm{C/m^2})t^2\}\,\hat{i}\ +$
$\{1 + [(1 \times 10^{-7}\ \mathrm{C})/4(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(1 \times 10^{-3}\ \mathrm{kg})](-5 \times 10^{-6}\ \mathrm{C/m^2})t^2\}\,\hat{j}$
$= \boxed{[(1 + 8.5t^2)\hat{i} + (1 - 14t^2)\hat{j}]\ \mathrm{m},\ \text{with } t \text{ in s}}$.

64. (a) At the origin the fields from the line charges are in opposite directions. From the symmetry of the charges, we see that the total field at the origin is

$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 = \boxed{0}.$$



(b) The force on a charge at the origin is

$$\vec{F}_0 = q\vec{E}_0 = \boxed{0}.$$

(c) We find the angles and distances for each line charge:

$\tan\theta_1 = 3/(4 - 1)$, which gives $\theta_1 = 45.0°$;

$\tan\theta_2 = 3/(4 + 1)$, which gives $\theta_2 = 31.0°$;

$r_1^2 = (3\text{ cm})^2 + (3\text{ cm})^2$, which gives $r_1 = 4.24$ cm;

$r_2^2 = (5\text{ cm})^2 + (3\text{ cm})^2$, which gives $r_2 = 5.83$ cm.

The total field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$
$$= (\lambda/2\pi\varepsilon_0 r_1)(\cos\theta_1\,\hat{i} - \sin\theta_1\,\hat{j}) + (\lambda/2\pi\varepsilon_0 r_2)(\cos\theta_2\,\hat{i} - \sin\theta_2\,\hat{j})$$
$$= (\lambda/2\pi\varepsilon_0)\{[(\cos\theta_1\,\hat{i} - \sin\theta_1\,\hat{j})/r_1] + [(\cos\theta_2\,\hat{i} - \sin\theta_2\,\hat{j})/r_2]\}$$
$$= [(5\times10^{-6}\text{ C/m})/2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)]$$
$$[(\cos 45.0°\,\hat{i} - \sin 45.0°\,\hat{j})/(4.24\times10^{-2}\text{ m}) + (\cos 31.0°\,\hat{i} - \sin 31.0°\,\hat{j})/(5.83\times10^{-2}\text{ m})]$$
$$= \boxed{(2.8\times10^6\text{ N/C})\hat{i} - (1.5\times10^6\text{ N/C})\hat{j}}.$$

The force on the charge is

$$\vec{F} = q\vec{E} = (6\times10^{-6}\text{ C})[(2.8\times10^{-6}\text{ N/C})\hat{i} - (1.5\times10^{-6}\text{ N/C})\hat{j}]$$
$$= \boxed{(17\hat{i} - 9\hat{j})\text{ N}}.$$

65. (a) The electric field of the plate is perpendicular to and away from the plate. The force on the positive charge is away from the plate:

$$F = qE = q\sigma/2\varepsilon_0$$
$$= (1.6\times10^{-19}\text{ C})(8.0\times10^{-6}\text{ C/m}^2)/2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)$$
$$= \boxed{7.2\times10^{-14}\text{ N away from the plate}}.$$

(b) We find the work from the work-energy theorem:

$$W = \Delta K$$
$$= 0 - (2\times10^6\text{ eV})(1.6\times10^{-19}\text{ J/eV}) = \boxed{-3.2\times10^{-13}\text{ J}}.$$

(c) Because the work is done by the electric field, we have

$$W = -Fd$$
$$-3.2\times10^{-13}\text{ J} = -(7.2\times10^{-14}\text{ N})d,\text{ which gives } d = \boxed{4.4\text{ m}}.$$

66. (a) When there is no charge on the drop, the forces acting on the drop are the downward force of gravity and the upward drag force;

$$mg - F_{\text{drag}} = 0,\quad\text{or}$$
$$\rho(\tfrac{4}{3}\pi r^3)g = 6\pi\eta r v_0,\text{ which gives } v_0 = 2r^2\rho g/9\eta.$$

(b) With a positive charge, the electric force is up, so we have

$$mg - F_{\text{drag}} - qE = 0,\quad\text{or}$$
$$qE = mg - F_{\text{drag}} = \rho(\tfrac{4}{3}\pi r^3)g - 6\pi\eta r v_1.$$

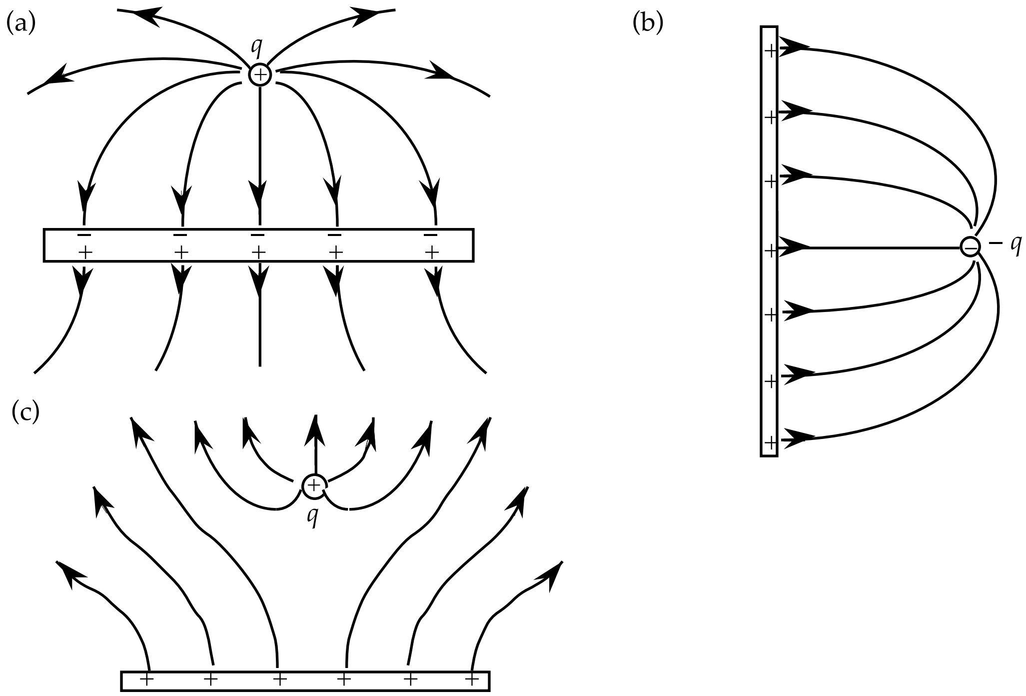
If we use the result of part (a), we get

$$qE = 6\pi\eta r v_0 - 6\pi\eta r v_1 = 6\pi\eta(v_0 - v_1)r = 6\pi\eta(v_0 - v_1)(9v_0\eta/2\rho g)^{1/2},\text{ which gives}$$
$$q = [18\pi(v_0 - v_1)/E](v_0\eta^3/2\rho g)^{1/2}.$$

(c) Because the droplet is stationary, there is no drag force, so we have

$$qE = mg = \rho(\tfrac{4}{3}\pi r^3)g;$$
$$E = \tfrac{4}{3}\pi r^3\rho g/q = \tfrac{4}{3}\pi(2.0\times10^{-6}\text{ m})(0.85\times10^3\text{ kg/m}^3)(9.8\text{ m/s}^2)/(1.6\times10^{-19}\text{ C}) = \boxed{1.74\times10^6\text{ N/C}}.$$

67.



68. We write the relation as

$t = \lambda^{\alpha}q^{\beta}m^{\gamma}R^{\delta}\varepsilon_0^{\mu}$. When we substitute the dimensions, we get

$[t] = [\lambda]^{\alpha}[q]^{\beta}[m]^{\gamma}[R]^{\delta}[\varepsilon_0]^{\mu}$;

$[T] = [QL^{-1}]^{\alpha}[Q]^{\beta}[M]^{\gamma}[L]^{\delta}[Q^2T^2M^{-1}L^{-3}]^{\mu}$.

We equate the exponents for each dimension:

Q: $0 = \alpha + \beta + 2\mu$;

L: $0 = -\alpha + \delta - 3\mu$;

M: $0 = \gamma - \mu$;

T: $1 = 2\mu$.

We have four equations with five unknowns. Two we can find directly:

$\mu = \frac{1}{2}$, $\gamma = \frac{1}{2}$.

To find the others we must use the fact that the field of a line charge depends on λ/ε_0. This is the only contributor to the force, so we expect the exponent of λ to be the negative of the exponent of ε_0:

$\alpha = -\mu = -\frac{1}{2}$; which gives $\beta = -\frac{1}{2}$, and $\delta = 1$.

When we use these results, we have

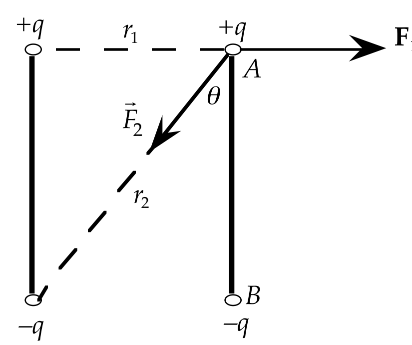
$\boxed{t \propto R(m\varepsilon_0/q\lambda)^{1/2}}$.

69. By symmetry the net force exerted on one rod by the other is perpendicular to each rod. Also, the forces from the other rod on the two charges at points A and B are the same. So the force between the two rods is

$F = 2(F_1 - F_2 \sin\theta) = 2[kq^2/r_1^2 - (kq^2/r_2^2)(r_1/r_2)]$

$= 2(9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(2.2 \times 10^{-4}\ \mathrm{C})^2\{1/(0.18\ \mathrm{m})^2 - (0.18\ \mathrm{m})/[(0.18\ \mathrm{m})^2 + (0.20\ \mathrm{m})^2]^{3/2}\}$

$= \boxed{1.9 \times 10^4\ \text{N, directly away from each other}}$.



The net torque on each rod is $\boxed{\text{zero}}$, since each of the two charges on each rod receives the same amount of force and is equidistant form the axis.

70. Choose an xy coordinate system originated at the point where the electron enters the region, with the x-axis pointing east and y-axis pointing north. The motion in the x-direction is uniform, so

$x = v_x t = (v_0 \cos\theta)t$, where $v_0 = 3 \times 10^6$ m/s and $\theta = 40°$.

In the y-direction there is an acceleration:

$a_y = F_y/m = -eE/m$, so

$y = v_{0y} t + \frac{1}{2}a_y t^2 = (v_0 \sin\theta)t + \frac{1}{2}(-eE/m)t^2$.

When the electron strikes the bottom plate $y = 0$, whereupon

$t = 2mv_0 \sin\theta/eE$. Plug this into the expression for x to obtain

$x = (v_0\cos\theta)(2mv_0\sin\theta/eE) = \boxed{mv_0^2 \sin(2\theta)/eE}$.

Note that this is analogous to the range formula for a projectile, with g replaced by eE/m.

71. We are given the force

$$\vec{F} = \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left\{\ln\left[\frac{R-(L/2)}{R+(L/2)}\right] + R\left[\frac{1}{R-(L/2)} - \frac{1}{R+(L/2)}\right]\right\}\hat{i}.$$

If we change variable to $x = L/2R$, the magnitude of the force becomes

$$F = \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left[\ln\left(\frac{1-x}{1+x}\right) + \left(\frac{1}{1-x} - \frac{1}{1+x}\right)\right] = \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left[\ln(1-x) - \ln(1+x) + \left(\frac{1}{1-x} - \frac{1}{1+x}\right)\right].$$

Using the approximate expansions for small x, we get

$$F = \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left[\left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots\right) - \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\right) + \left(1 + x + x^2 + x^3 + \ldots\right) - \left(1 - x + x^2 - x^3 + \ldots\right)\right]$$

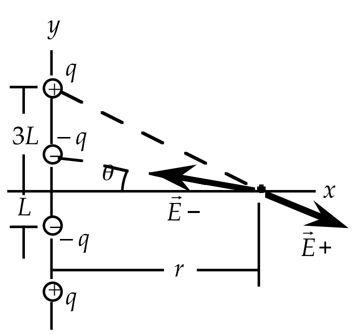
$$= \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left[\left(-2x - \frac{2x^3}{3} - \ldots\right) + \left(2x + 2x^3 + \ldots\right)\right] \approx \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left(\frac{4x^3}{3}\right).$$

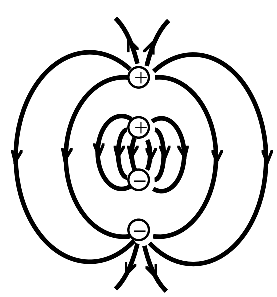
In terms of the distance R, the force is

$F = (q\lambda_0/2\pi\varepsilon_0 L)(4/3)(L/2R)^3 = q\lambda_0 L^2/12\pi\varepsilon_0 R^3$.

The field of a dipole on the axis is $E = p/2\pi\varepsilon_0 R^3$, so the dipole moment is $p = \boxed{\lambda_0 L^2/6}$.

72. (a) (b)





We orient the four charges along the y-axis, as shown in the diagram. From the symmetry of the charge distribution, we see that the resultant field will be along the x-axis, and we can double the difference between the component from the positive charge and that from the negative one. With r the distance along the x-axis, we have

$\vec{E} = 2(E_{+x} - E_{-x})\hat{i} = 2(q/4\pi\varepsilon_0)[\cos\theta_+/[r^2 + (3L)^2] - \cos\theta_-/(r^2 + L^2)]\hat{i}$

$= (q/2\pi\varepsilon_0)\{r/[r^2 + (3L)^2]^{3/2} - r/(r^2 + L^2)^{3/2}\}\hat{i}$.

If we use the approximation given in the hint, we get

$\vec{E} = (qr/2\pi\varepsilon_0)[1/r^3 - 3(3L)^2/2r^5 - 1/r^3 + 3L^2/2r^5]\hat{i} = -(6qL^2/\pi\varepsilon_0 r^4)\hat{i}$.